Stochastic Models of Quality in Continuous Information Systems

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Abstract. Mathematical models of main stochastic characteristics of continuous multi-stage inter-operational control for the IS of mechatronics were created, when the flows of rejected IS are returned to the manufacture process for regeneration, and IS classification rules are analogous at the separate control stages, and when undeniable IS classification errors of the first and second kind are present. The efficiency of control operation is estimated in the way of modeling according to transformations of density functions of defectivity level at separate stages, by applying beta distribution and dynamics of defectivity level mean variation. It was shown that returning flows are determined by error probabilities and the level of defectivity before control, and the models of separate stages differ only in the magnitude of generalized error, which equals the product of generalized error probabilities of separate stages. Manufacturing peculiarities of IS and their components were employed during creation of models.

Keywords. Information systems, IS, efficiency, models, quality, beta-distribution.

1. Introduction

In companies which utilize modern technologies databases of manufactured production and technological processes are constantly maintained and complemented. Data is often read and input automatically. By using computer networks and database control and analysis systems, information can be transmitted in real-time and can be used in decision making processes at each intermediate or final stage of manufacture. Thus there is a possibility to use information not only from the current but also from the previous stages of manufacture. That can increase the quality of information systems (IS) [1].

Modern IS control system involves control of raw materials, components, transitional control of elements or nodes, technological process and final product. In this way IS control system plays an important role by influencing cost and the final result. When control is tightened (when new control stages are introduced, number of controlled parameters is increased, rules of acceptance are changed, etc.) losses due to reclamations decrease, but internal losses will grow due to increased control work rate and due to the fact, that more quality products will be falsely ranked as defective. On the other hand, when control is loose, losses due to reclamations will increase, but internal losses will decrease. Naturally a task emerges to select economically optimal variant of control. It is obtained by minimizing adequately-formulated function of losses.

Tasks solved during control are attributed to several areas of mathematical statistics, specifically – decision making theory, identification, classification, discriminational analysis, image recognition, etc. But in manufacture it is apparently more convenient (compare to other areas) to apply methods of mathematical statistics, for which apriori information is required.

Continuous inter-operational quality control is commonly applied in manufacture process of IS products. Selective control peculiarities of such IS are analyzed in publications [1 – 6] on the grounds of color picture tube manufacture specifics. In this paper we will describe the performance of multistage continuous inter-operational control with the help of stochastic models, when IS classification errors of the first and second kind are present. Main attention is paid to the transformation of production defectivity level probability distributions, which in turn allows to estimate the efficiency of inter-operational control in the way of modeling, and to select the required number of control stages and their characteristics.

Complex mechatronics products are defined in technical documentation as an entire series of parameters, the values of which determine the level of product quality. Parameters can be
differentiated according to their importance regarding the implementation of purpose functions. International standard ISO-2859-0 [11] recommends to divide parameters into two – A, B – or three – A, B, C – classes (groups). Here A – most important or significant parameters, and B, C – secondary or less significant parameters. Such classification of parameters is convenient when analyzing problems of multiparametric product quality control [7–10]. When imitational modeling is applied, stochastic models of quality level are required for separate parameters, their groups and for entire product.

2. Control scheme and models

Inter-operational control fragment is presented in Fig. 1, which involves two stages of continuous control \( K_1 \) and \( K_2 \) (in both stages IS are classified according to analogical decision rules).

There is a probability \( \omega \) that a defective IS after manufacture operations G will enter the control stage \( K_1 \) which is characterized by classification error probabilities \( \alpha_1 \) and \( \beta_1 \); probability \( \omega \) is transformed to parameter \( \theta \) after control operations. IS acceptance probability is \( p_1 \) and rejection probability is \( q_1 \).

Analogously in the second stage \( K_2 \) with errors \( \alpha_2 \) and \( \beta_2 \) parameter \( \theta \) is transformed to \( \tau \), when probabilities \( p_2, q_2 \) are fixed. IS in such scheme is accepted with probability \( p_{12} \). Rejected IS are returned for reparation to manufacture process G.

3. Models with random values

The defective IS probability in the entirety of IS, as a random value [2], before control \( K_1 \) is characterized by density \( f(\omega) \) (Fig. 2), before \( K_2 \) – by density \( g(\theta) \) and after control \( K_2 \) – by density \( h(\tau) \).

The means of defectivity level respectively are \( \mu_\omega, \mu_\theta, \mu_\tau \):

\[
\begin{align*}
\mu_\omega &= \int_0^1 \omega f(\omega) d\omega, \\
\mu_\theta &= \int_0^1 \theta g(\theta) d\theta, \\
\mu_\tau &= \int_0^1 \tau h(\tau) d\tau.
\end{align*}
\]

We receive average sizes of accepted flows \( \bar{p}_1, \bar{p}_2, \bar{p}_{12} \), and average flows of rejected products \( \bar{q}_1, \bar{q}_2, \bar{q}_{12} \):

\[
\begin{align*}
\bar{p}_1 &= (1 - \alpha_1)(1 - (1 - \beta_1)\mu_\omega), \\
\bar{p}_2 &= (1 - \alpha_2)(1 - (1 - \beta_2)\mu_\theta), \\
\bar{p}_{12} &= \bar{p}_1\bar{p}_2; \\
\end{align*}
\]

where \( \bar{q}_1 = 1 - \bar{p}_1 \), \( \bar{q}_2 = 1 - \bar{p}_2 \), \( \bar{q}_{12} = 1 - \bar{p}_{12} \).

Densities \( f(\omega), g(\theta), h(\tau) \) are presented in Fig. 3.
4. Model with two – stage control

Let’s consider two-stage continuous control (modified), in which the returning flow is passed into its own repair (regeneration) operation $R_i$ after each control stage $K_i$, $i=1, 2$ (Fig. 4).

$$g(\theta), h_i(\tau_i), h^*_i(\tau^*_i), i=1, 2; \quad \beta_0 = \bar{\beta} = \frac{1}{2}$$

Figure 4. The modified two-stage continuous control

Densities $h(\tau)$ and $h_2(\tau_2)$ are shown in Fig. 5.

$$\eta_i = 1 - \theta_i.$$ (4)

R.q. $\eta_i$ is characterized by density $\varphi_i(\eta_i)$, distribution function $\Phi_i(\eta_i)$, mean $E\eta_i = \mu_i$ and dispersion $V\eta_i = V\theta_i = \sigma^2_i$. The following relation formulas are valid:

$$\begin{aligned}
\mu_i &= 1 - \mu_i, \\
g_i(\theta_i) &= \varphi_i(1 - \theta_i), \\
G_i(\theta_i) &= 1 - \Phi_i(1 - \theta_i).
\end{aligned}$$ (5)

The probability of defective product for the entire product r.q. $\theta$ and probability of good product r.q. $\eta$ are calculated as

$$\eta = \prod_{i=1}^{j} \eta_i \quad \quad \theta = 1 - \eta = 1 - \prod_{i=1}^{j} (1 - \theta_i).$$ (6)

Since $\eta_i$ are inter-independent r.q., the following equation is valid

$$\varphi(\eta) = \prod_{i=1}^{j} \varphi_i(\eta_i).$$ (7)

In general, the distribution function $\Phi(\eta)$ of the random multiplicative r.q. $\eta = \eta_1 \ldots \eta_l$ according to [8,9] is defined by 1-dimensional integral

$$\Phi(\eta) = \int_{D_\eta} \prod_{i=1}^{j} \phi_i(\eta_i)\varphi_2(\eta_2) \ldots \varphi_l(\eta_l) d\eta_1 d\eta_2 \ldots d\eta_l,$$ (8)

here $D_\eta$ – integration range. Then density $\varphi(\eta)$ is expressed according to (9)

$$\varphi(\eta) = \frac{d\Phi(\eta)}{d\eta}.$$ (9)

We will provide the main formulas, required for the further analysis, when r.q. $\theta_i$ and also $\eta_i$ are distributed according to the beta-distribution with shape parameters $a_i, b_i$ and marking that:

$$\begin{aligned}
\theta_i &\sim B(e(a_i, b_i)), \\
\eta_i &\sim B(e(b_i, a_i)).
\end{aligned}$$ [8]:

$$\begin{aligned}
g_i(\theta_i) &= B_i^{-1}(a_i, b_i) \theta_i^{a_i-1}(1 - \theta_i)^{b_i-1}, \\
\varphi_i(\eta_i) &= B_i^{-1}(a_i, b_i) \eta_i^{b_i-1}(1 - \eta_i)^{a_i-1},
\end{aligned}$$ (10)

here $B_i(a_i, b_i) = \frac{\Gamma(a_i) \Gamma(b_i)}{\Gamma(a_i + b_i)}$ – beta function, $\Gamma(\cdot)$ – gamma function, $\Gamma(z_i) = (z_i-1)\Gamma(z_i-1)$.
or $\Gamma(n) = (n-1)!$, when $n$ is a whole number (h.n.), $i = 1, \ldots, l$;

$$
\mu_i = \frac{a_i}{a_i + b_i}, \quad \overline{\mu_i} = 1 - \mu_i = \frac{b_i}{a_i + b_i},
$$
and numerical

$$
\sigma_i^2 = \frac{a_i b_i}{(a_i + b_i)^2(a_i + b_i + 1)} = \frac{\mu_i}{a_i + b_i + 1}.
$$

Then according to (7), $\eta_i = \mu_i$.

Density $g_i(\theta_t)$ has maximum at the point $\theta_{iM}$ (mode) [8]

$$
\theta_{iM} = \frac{a_i - 1}{a_i + b_i - 2}.
$$

Respectively the maximum of density $\varphi_i(\eta_i)$ is at the point $\eta_{iM} = 1 - \theta_{iM}$.

It is obvious, that (9)–(13) formulas may be applied for entire product, if r.q. $\theta \sim Be(a, b)$, or for separate groups $A$, $B$, $C$, if their defectivity levels $\theta_A$, $\theta_B$, $\theta_C$ are characterized by beta-distribution.

IS is characterized using two parameters $i = 1, 2$, which are distributed according to beta-distribution (4 – 14). Then according to (7), equations $\eta = \eta_1 \eta_2$, $\theta = \theta_1 + \theta_2 - \theta_1 \theta_2$ are valid.

In order to avoid integration bands $\eta_i = 0$ we use dependency

$$
F(y) = P[Y < y] = 1 - P[Y > y],
$$
here $F(y)$ is distribution function of r.q. $Y$, $P[Y > y]$ is probability, that $Y > y$. In this way, when $Y > y$, integration range $D_\eta$ (9) is defined by hyperbola $\eta = \eta_1 \eta_2$ with upper variation interval limit $\eta_1 = 1$ according to [1] (Fig. 6).

![Figure 6. Two-dimensional integration space](image)

We receive

$$
\Phi(\eta) = 1 - \int_{\eta} \varphi_1(\eta_1) d\eta_1 \int_{\eta} \varphi_2(\eta_2) d\eta_2 =
$$

$$
= 1 - (B_2B_2)^{-1} \int_{\eta} \int_{\eta} \eta_1^{\eta_1-1} \eta_2^{\eta_2-1} d\eta_1 d\eta_2,
$$

$$
\varphi(\eta) = d\Phi(\eta) \overline{d\eta} = \int_{\eta} \varphi_1(\eta_1) \varphi_2 \left( \eta \overline{\eta} \right) d\eta_1 =
$$

$$
= 1 - (B_2B_2)^{-1} \int_{\eta} \int_{\eta} \eta_1^{\eta_1-1} \eta_2^{\eta_2-1}.
$$

In order to obtain beta-densities in groups and for entire product, the condition $b_1 > b_2 \ldots > b_l$ according to (16) must be satisfied. Since $a = a_i = 10$, when $\mu = \varpi = 0.5$ we have $b = b_i = 10$ and $b_1 = b_0 + (10 - i)$, $i = 1 \ldots 9$. Then $b_9 = 11$, $b_8 = 12$, $b_7 = 13$, $b_6 = 14$, $b_5 = 15$, $b_4 = 16$, $b_3 = 17$, $b_2 = 18$, $b_1 = 19$.

Models of beta-densities according to (10) with their shape parameters: $\theta_t \sim (1, b_1)$, when $b_{10} = 10$, $b_1 = 19$. Average defectivity levels according to separate parameters:

$\mu_1 = 0.05$, $\mu_2 = 0.0526$, $\mu_3 = 0.0556$, $\mu_4 = 0.0588$, $\mu_5 = 0.0625$, $\mu_6 = 0.0667$, $\mu_7 = 0.0714$, $\mu_8 = 0.0769$, $\mu_9 = 0.0833$, $\mu_{10} = 0.0909$.

Densities $g_i(\theta_t)$ are monotonically decreasing functions from $g_i(0) = b_i$ to $g_i(l) = 0$.

Densities and main numerical characteristics according to groups for both cases (Table 6) are slightly different, but for entire product densities $g(\theta)$ of defectivity level $\theta$ and numerical characteristics $\mu$, $\sigma^2$ practically coincides for both cases. It is obvious, that coincidence of all characteristics is the better the smaller $\mu$ value is.

6. Practical applications

Three groups $A$, $B$, $C$; $\mu = 10\%$, $l = 10$, $r = 2$, $s = 3$, $A \in i = 1, 2$, $B \in i = 3, 4 \equiv j$, $C \in i = 6 + 10 \equiv k$; $a_j = 1$, $a_j = 2$, $a_k = 3$.

Modeling results are presented in Table 1. In columns $AB$, $BC$, $AC$ modeling results are indicated, when one of parameter groups is eliminated and IS is evaluated according to
remaining two groups. For variant AC the condition of exact beta-distribution between A and C groups \((b_2 = 228 > b_6 + a_6 = 222)\) is not met any more, therefore density \(g_{AC}(\theta_{AC})\) is approximated using density

\[
g_{\sigma}(\theta_{AC}) = \frac{\Gamma(224.6)\theta^{16}(1-\theta)^{206.6}}{16\Gamma(207.6)},
\]

where \(\theta_{AC} = 3.1 \times 10^4\)

and remaining densities are described as in \(m = 6\%\), \(l = 4\), \(\sigma = s = 2\), \(A \leq i = 1,2\), \(B \leq j = 3,4\), \(a_i = 1, a_j = 2\) with their parameters \(a, b\) (when \(\theta_M > 0\), densities are unimodal, and when \(\theta_M = 0\) – monotonically decreasing curve).

**Table 1.** \(l = 10\), \(\mu = 10\%\); groups A, B, C, \(r = 2\), \(s = 3\)

<table>
<thead>
<tr>
<th>(i)</th>
<th>(a)</th>
<th>(b)</th>
<th>(\theta)</th>
<th>(\mu)</th>
<th>(\sigma)</th>
<th>(\nu)</th>
<th>(\theta_{\mu})</th>
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<tr>
<td>1</td>
<td>2</td>
<td>229</td>
<td>229/230</td>
<td>0.435</td>
<td>0.432</td>
<td>0.435</td>
<td>0.435</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>228</td>
<td>228/229</td>
<td>0.437</td>
<td>0.435</td>
<td>0.435</td>
<td>0.435</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>226</td>
<td>226/228</td>
<td>0.877</td>
<td>0.461</td>
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</tr>
<tr>
<td>4</td>
<td>2</td>
<td>224</td>
<td>224/226</td>
<td>0.885</td>
<td>0.621</td>
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<tr>
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<td>2</td>
<td>222</td>
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<td>0.893</td>
<td>0.627</td>
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<td>0.627</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
<td>219</td>
<td>219/222</td>
<td>1.53</td>
<td>0.733</td>
<td>0.733</td>
<td>0.733</td>
</tr>
<tr>
<td>7</td>
<td>3</td>
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<td>216/219</td>
<td>1.57</td>
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<td>8</td>
<td>3</td>
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<td>1.89</td>
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<td>0.817</td>
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<td>A</td>
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<td>C</td>
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<td>1.97</td>
<td>1.97</td>
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<td>4255</td>
<td>7.57</td>
<td>1.76</td>
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</table>

*) density \(g_{AC}(\theta_{AC})\) is approximated using beta-density \(g_{\sigma}(\theta_{AC})\).

### 7. Conclusions

1. Received expressions give opportunity to model various desirable situations in the scheme of inter-operational control in visual manner, according to transformations of density of defectivity levels, by defining density parameters at desired point of scheme (density of beta distribution) and selecting real error probability values of separate control stages.

2. The control scheme with localized repair operations of rejected products (Fig. 3) is more efficient than the scheme in Fig. 2, in which the returning flows are returned back to the manufacture, but the maintenance of individual repair workplaces is more expensive from economical point of view.

3. It is purposeful to use approximation of density \(g(\theta)\) by beta-density \(g_{\sigma}(\theta)\) when number of parameters \(l < 5\), since when \(l\) value increases, the relative approximation error also increases.

4. Grouping of parameters according to their significance into classes (groups) A, B, C is recommended by ISO standards and it enables to highlight the influence of these groups onto the overall product defectivity level; it also enables modeling of situations when one group of parameters is eliminated or additional parameters are introduced.

### 7. References


